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Notes on Heterotic Compactifications

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In this technical note we describe a pair of results on heterotic compactifications. First, we give an example demonstrating that the usual statement of the anomaly-freedom constraint for perturbative heterotic compactifications (meaning, matching second Chern characters) is incorrect for compactifications involving torsion-free sheaves. Secondly, we correct errors in the literature regarding the counting of massless particles in heterotic compactifications.

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1 Introduction

Historically compactifications of heterotic string theory have been extremely difficult to study. Not only must one specify a Calabi-Yau variety to compactify on, but one must also specify at least one sheaf. Although in the last ten years we have come to a good understanding of a large number of Calabi-Yau's, the physics community still does not have a good grasp on compactifications that also involve sheaves.

There has been some limited progress in this direction, but it leaves much to be desired. For example, methods to study bundles on elliptic Calabi-Yau's [14, 15, 16, 17] have recently been developed, but unfortunately this does not give any insight into more general sheaves, or sheaves on Calabi-Yau's that are not elliptic. An older approach to the problem [4] constructs heterotic conformal field theories as infrared limits of gauged linear sigma models [3], but gives little understanding of moduli space questions.

This paper is one of a set [13] devoted to beginning to put our understanding of heterotic compactifications on footing as solid as that of type II compactifications. In particular, in this paper we correct two misconceptions in the literature. First we observe that the usual statement of anomaly freedom in perturbative heterotic compactifications is incorrect. Second, we correct misunderstandings in the literature regarding the counting of massless particles in geometric compactifications of heterotic string theory.

While our thesis advisor was reviewing a final draft of this paper, the work [23] appeared, which has overlap with the results in section 4.

2 A Rapid Review of Heterotic Compactifications

For a consistent perturbative compactification of either the $E_8 \times E_8$ or $Spin(32)/\mathbf{Z}_2$ heterotic string, in addition to specifying a Calabi-Yau Z one must also specify a set of Mumford-Takemoto semistable [1, 2], torsion-free sheaves¹ V_i . These sheaves must obey two constraints. One constraint² can be written as

$$\omega^{n-1} \cup c_1(V_i) = 0$$

where n is the complex dimension of the Calabi-Yau, and ω is the Kähler form. The other is an anomaly-cancellation condition which, if a single V_i is embedded in each E_8 , has his-

¹In this paper, by a locally free sheaf, we mean a bundle. A torsion-free sheaf is locally free up to complex codimension two, and a reflexive sheaf is locally free up to complex codimension three, on a smooth variety. For more information see [1, 2, 13].

²For example, for compactifications to four dimensions, N=1 supersymmetry, on a Calabi-Yau X one gets a D term in the low-energy effective action proportional to $\langle X | \omega^2 \cup c_1(V) \rangle$.

torically been written as

$$\sum_i \left(c_2(V_i) - \frac{1}{2} c_1(V_i)^2 \right) = c_2(TZ)$$

In section 3 we show that it is possible to have a consistent perturbative heterotic compactification in which this constraint is violated.

It was noted [10] that the anomaly-cancellation conditions can be modified slightly by the presence of five-branes in the heterotic compactification. Let $[W]$ denote the cohomology class of the five-branes, then the second constraint above is modified to

$$\sum_i \left(c_2(V_i) - \frac{1}{2} c_1(V_i)^2 \right) + [W] = c_2(TZ)$$

In this technical note we will only be concerned with perturbative heterotic compactifications.

Historically, for a long time the only perturbative heterotic compactifications studied were those in which one took $V = TZ$, the “standard embedding.” This was done partly because more general compactifications are more difficult to work with, and partly because it was believed more general compactifications were destabilized by worldsheet instantons [5]. For the (0,2) models of [4], both difficulties have been overcome [4, 6].

3 Anomaly-free compactifications

In this section we will give an example of an anomaly-free perturbative heterotic compactification on a Calabi-Yau Z involving a sheaf \mathcal{E} of $c_1(\mathcal{E}) = 0$ such that $c_2(\mathcal{E}) \neq c_2(TZ)$.

Before we begin, we should explain how this is possible. There are two ways one can check that a perturbative heterotic compactification is anomaly-free. One way is to check that the worldsheet conformal field theory has no chiral anomalies. The other way is through a constraint in the low-energy supergravity of the form

$$dH = \text{tr} R \wedge R - \text{tr} F \wedge F \tag{1}$$

Ordinarily both methods yield the same result:

$$c_2(\mathcal{E}) = c_2(TZ)$$

However, there is a subtlety. Constraint (1) is only sensibly defined when \mathcal{E} is a bundle. Only in that case can one define a connection, and thereby make sense of F . When \mathcal{E} is a more general coherent sheaf, one can not define a connection, and so there is no way to assign a meaning to F . In that case, the only way to check for anomalies is to study the worldsheet

conformal field theory. In particular, below we will give an example of an anomaly-free nonsingular conformal field theory describing a torsion-free sheaf \mathcal{E} in which $c_1(\mathcal{E}) = 0$ and $c_2(\mathcal{E}) \neq c_2(TZ)$.

The specific example we will consider is of a sheaf \mathcal{E} on an elliptic threefold Z (fibered over the Hirzebruch surface \mathbf{F}_1). The conformal field theory is constructed using a gauged linear sigma model [3], in the form of a (0,2) model of Distler, Kachru [4]. We shall assume the reader is familiar with the material in [3, 4].

More precisely, the Calabi-Yau Z is realized as a hypersurface in an ambient space X which is a \mathbf{P}^2 fibered over \mathbf{F}_1 . This ambient space X can be described as the quotient³

$$X = \frac{\mathbf{C}^7 - S_{exc}}{(\mathbf{C}^\times)^3}$$

in other words, as homogeneous coordinates modded out by \mathbf{C}^\times actions, where the homogeneous coordinates u, v, w, s, x, y, z have weights under the \mathbf{C}^\times actions λ, μ, ν as

	u	v	w	s		x	y	z
λ	1	1	1	0		6	9	0
μ	0	0	1	1		4	6	0
ν	0	0	0	0		1	1	1

The “exceptional set” S_{exc} is a set of points to be omitted from \mathbf{C}^7 before quotienting. This set depends upon the precise phase of X , in the language of [3], or more abstractly, S_{exc} determines the precise representative of the birational equivalence class. We will only consider the case that the Calabi-Yau hypersurface is a $K3$ -fibration, in which event S_{exc} is

$$S_{exc} = \{u = v = 0\} \cup \{w = s = 0\} \cup \{x = y = z = 0\}$$

We will (naively) specify the sheaf \mathcal{E} as the (restriction to the hypersurface of a) kernel of a short exact sequence, defined on X as

$$0 \longrightarrow \mathcal{E} \longrightarrow \mathcal{O}(1, 0, 1)^3 \oplus \mathcal{O}(1, 1, 0) \oplus \mathcal{O}(24, 3, 14) \xrightarrow{F_a} \mathcal{O}(28, 4, 17) \longrightarrow 0 \quad (2)$$

In the present notation, $c_1(\mathcal{O}(a, b, c)) = aD_u + bD_z + cD_s$, where for example⁴ $D_u = \{u = 0\}$.

³The technically astute reader will recognize that this space is a toric variety, and can be constructed as a fan with edges

$$\begin{aligned} \nu_u &= (1, 0, 0, 0), \nu_v = (-1, -1, -6, -9), \nu_w = (0, 1, 0, 0), \nu_s = (0, -1, -4, -6) \\ \nu_x &= (0, 0, 1, 0), \nu_y = (0, 0, 0, 1), \nu_z = (0, 0, -1, -1) \end{aligned}$$

⁴We are maliciously failing to distinguish between divisor classes and elements of $H^2(\mathbf{Z})$.

We need to check that the (0,2) model defined above is anomaly-free and defines a nonsingular conformal field theory. To see that it is anomaly-free, we compute the chiral anomalies associated with each $U(1)$ in the linear sigma model.

The linear sigma model contains a (0,2) chiral superfield for each homogeneous coordinate, and in addition an auxiliary superfield we shall label p , with charges under the $U(1)$'s (which we shall label as λ, μ, ν , for obvious reasons) as follows

	u	v	w	s	x	y	z	p
λ	1	1	1	0	6	9	0	-28
μ	0	0	1	1	4	6	0	-17
ν	0	0	0	0	1	1	1	-4

In addition, the linear sigma model contains Fermi superfields $\Lambda_1, \Lambda_2, \dots, \Lambda_5, \Sigma$, with $U(1)$ charges

	Λ_1	Λ_2	Λ_3	Λ_4	Λ_5	Σ
λ	1	1	1	1	24	-18
μ	1	1	1	0	14	-12
ν	0	0	0	1	3	-3

The (0,2) chiral superfields each contain a right-moving fermion, and the Fermi superfields each contain a left-moving fermion. Schematically, if we let $q_{+i}^{(k)}$ denote the charge of the i th right-moving fermion with respect to the k th $U(1)$, and similarly for $q_{-i}^{(k)}$, then the anomaly cancellation condition can be written as

$$\sum_i q_{+i}^{(j)} q_{+i}^{(k)} = \sum_i q_{-i}^{(j)} q_{-i}^{(k)}$$

for all pairs j, k . It is straightforward to check that the linear sigma model defined above is anomaly-free. Ordinarily this result would imply⁵ $c_2(\mathcal{E}) = c_2(TZ)$, however we shall see momentarily that in the present case this is false.

In fact we have been quite naive in equation (2). It is easy to check that each of the maps F_a necessarily vanishes at $x = z = 0$, so the short exact sequence shown is actually not exact. This problem and its solution were first described in [9]. The sequence shown can be corrected to an exact sequence by adding a term as

$$0 \rightarrow \mathcal{E} \rightarrow \mathcal{O}(1, 0, 1)^3 \oplus \mathcal{O}(1, 1, 0) \oplus \mathcal{O}(24, 3, 14) \rightarrow \mathcal{O}(28, 4, 17) \rightarrow \mathcal{O}_S(28, 4, 17) \rightarrow 0$$

⁵In fact, demanding anomaly-freedom is somewhat stronger [9], as it (ordinarily) demands matching Chern characters in each possible phase of the linear sigma model. In order to match Chern characters in each phase, linear relations that exist between divisors can not be taken into account.

where $\mathcal{O}_S(28, 4, 17)$ is a skyscraper sheaf localized over the subvariety $S = \{x = z = 0\}$. Since S is codimension two, \mathcal{E} is a torsion-free sheaf on Z .

Often when all of the maps F_a in a short exact sequence such as (2) vanish, the associated conformal field theory becomes singular. In the linear sigma model, this can be seen as a boson gaining a noncompact flat direction. Here, however, this does not happen – on the subvariety S , none of the linear sigma model's bosons gain noncompact flat directions. (This phenomenon was also first discussed in [9].) This can be seen by examining the D terms for the linear sigma model describing this (0,2) model. The three D -term constraints⁶ are

$$\begin{aligned} |u|^2 + |v|^2 + |w|^2 + 6|x|^2 + 9|y|^2 - 28|p|^2 &= r_1 \\ |w|^2 + |s|^2 + 4|x|^2 + 6|y|^2 - 17|p|^2 &= r_2 \\ |x|^2 + |y|^2 + |z|^2 - 4|p|^2 &= r_3 \end{aligned}$$

where the r_i are Fayet-Iliopoulos terms. Over $S = \{x = z = 0\}$ these constraint equations can be rewritten as

$$|u|^2 + |v|^2 - |s|^2 + |p|^2 = r_1 - r_2 - 3r_3 \quad (3)$$

$$|w|^2 + |s|^2 + 7|p|^2 = r_2 - 6r_3 \quad (4)$$

$$|y|^2 - 4|p|^2 = r_3 \quad (5)$$

The $K3$ -fibration phase is the region

$$r_1 - r_2 - 3r_3 > 0$$

$$r_2 - 6r_3 > 0$$

$$r_3 > 0$$

The only boson whose vacuum expectation value is in danger of becoming unbounded over S is p , and it should be clear from equation (4) that the vacuum expectation value of p is bounded.

So far we have found an example of a conformal field theory that is nonsingular and anomaly-free, describing a torsion-free sheaf over a Calabi-Yau. Although the conformal field theory is anomaly-free, we now claim that $c_1(\mathcal{E}) = 0$ and $c_2(\mathcal{E}) \neq c_2(TZ)$.

The total Chern class of \mathcal{E} can be calculated as

$$c(\mathcal{E}) = \frac{c[\mathcal{O}(1, 0, 1)^3 \oplus \mathcal{O}(1, 1, 0) \oplus \mathcal{O}(24, 3, 14)]}{c[\mathcal{O}(28, 4, 17)]} c[\mathcal{O}_S(28, 4, 17)]$$

The skyscraper sheaf contributes to the second Chern class as

$$c_2(\mathcal{E}) = c_2(TZ) - 6D_u \cdot D_z - 4D_s \cdot D_z - D_z^2$$

⁶We are maliciously failing to distinguish chiral superfields, their bosonic components, and the vacuum expectation values of the bosonic components.

In particular, in the $K3$ -fibration phase $H^4(X, \mathbf{Z})$ is generated by $D_u \cdot D_s$, $D_u \cdot D_z$, $D_s \cdot D_z$, and D_z^2 , so this contribution is necessarily nonzero.

Finally, note that the subvariety $S = \{x = z = 0\}$ is located on the Calabi-Yau hypersurface (in fact, is the section of the elliptic fibration), and that for generic complex structure the Calabi-Yau is smooth. Thus, the restriction of \mathcal{E} to the Calabi-Yau defines a torsion-free sheaf on the Calabi-Yau, such that $c_1(\mathcal{E}) = 0$ and $c_2(\mathcal{E}) \neq c_2(TZ)$, and the associated conformal field theory is anomaly-free and nonsingular.

In principle, we should also check stability of \mathcal{E} , but unfortunately except for a few consistency checks (see [11] for a recent discussion), it is not known how to do this for $(0,2)$ models, so we shall simply ignore this difficulty. Also, $(0,2)$ models sometimes suffer from certain additional poorly-understood anomalies [8]. In the event our particular example suffered from either difficulty, we are completely confident other, better behaved, examples could be found.

4 Massless modes in heterotic compactifications

As is well-known, (geometric) perturbative string compactifications have massless modes corresponding to complex and Kahler deformations of the Calabi-Yau. In (geometric) heterotic compactifications there are additional charged and neutral massless modes, which have been historically mapped to certain sheaf cohomology groups [7]. In this section we will demonstrate that for heterotic compactifications involving coherent sheaves which are not locally free, the standard lore must be modified – the sheaf cohomology groups must be replaced with Ext groups.

We will assume the reader has some basic familiarity with sheaf theoretic homological algebra, as for example can be gained by reading the second appendix of [12]. To review briefly, for any coherent sheaves \mathcal{E}, \mathcal{F} , there are two sheaf-theoretic versions of Ext. One is a sheaf, known as (local) $Ext^i(\mathcal{E}, \mathcal{F})$. The other is a group, known as (global) $Ext^i(\mathcal{E}, \mathcal{F})$. They are related to one another by spectral sequences: the group (global) Ext is the limit of either of the spectral sequences with second level terms

$$\begin{aligned} E_2^{p,q} &= H^p(Ext^q(\mathcal{E}, \mathcal{F})) \\ E_2^{'p,q} &= H^q(Ext^p(\mathcal{E}, \mathcal{F})) \end{aligned}$$

First, it is often stated in the physics literature that deformations of a sheaf \mathcal{E} are in one-to-one correspondence with elements of the sheaf cohomology group $H^1(End\mathcal{E})$. However, this statement is strictly correct only when \mathcal{E} is locally free. More generally, when \mathcal{E} is some arbitrary coherent sheaf, deformations of \mathcal{E} are in one-to-one correspondence with elements of (global) $Ext^1(\mathcal{E}, \mathcal{E})$.

The group $\text{Ext}^1(\mathcal{E}, \mathcal{E})$ receives contributions from both $H^1(\text{End}\mathcal{E})$ as well as $H^0(\text{Ext}^1(\mathcal{E}, \mathcal{E}))$. When \mathcal{E} is locally free, the sheaf $\text{Ext}^1(\mathcal{E}, -) = 0$, and so in this special case $\text{Ext}^1(\mathcal{E}, \mathcal{E}) = H^1(\text{End}\mathcal{E})$.

This fact has consequences for the other massless modes. For example, consider embedding a rank 3 torsion-free sheaf \mathcal{E} in an E_8 , breaking it to E_6 . All massless modes in the compactification arise from deformations of the ten-dimensional adjoint E_8 vector, so as we have the group-theoretic decomposition of the **248** of E_8 into representations of $SU(3) \times E_6$ as

$$\mathbf{248} = (\mathbf{1}, \mathbf{78}) \oplus (\mathbf{3}, \mathbf{27}) \oplus (\overline{\mathbf{3}}, \overline{\mathbf{27}}) \oplus (\mathbf{8}, \mathbf{1})$$

we have massless scalars in E_6 representations as

Group	E_6 representation
$\text{Ext}^1(\mathcal{O}, \mathcal{O})$	78
$\text{Ext}^1(\mathcal{O}, \mathcal{E})$	27
$\text{Ext}^1(\mathcal{E}, \mathcal{O})$	$\overline{27}$
$\text{Ext}^1(\mathcal{E}, \mathcal{E})$	1

(Note that $\text{Ext}^1(\mathcal{O}, \mathcal{O}) = h^{1,0} = 0$ on a Calabi-Yau n -fold for $n > 1$, so in these cases there are no scalars in the **78** (only a vector).) In the existing literature it is often incorrectly claimed these massless modes are counted by sheaf cohomology groups; here we see the correct counting involves Ext groups instead. (In the special case that \mathcal{E} is locally free, the Ext groups simplify to sheaf cohomology groups.)

Many of these Ext groups are equal to sheaf cohomology groups for arbitrary coherent \mathcal{E} . For example⁷ [22, section III.6], on any variety X , for all coherent sheaves \mathcal{E} ,

$$(\text{local}) \text{Ext}^i(\mathcal{O}_X, \mathcal{E}) = \begin{cases} \mathcal{E} & i = 0 \\ 0 & i > 0 \end{cases}$$

so in particular

$$(\text{global}) \text{Ext}^i(\mathcal{O}_X, \mathcal{E}) \cong H^i(X, \mathcal{E}) \forall i \geq 0$$

Another useful fact [22, section III.6] is that if \mathcal{L} is any locally free sheaf and \mathcal{F}, \mathcal{G} are any coherent sheaves, then

$$(\text{local}) \text{Ext}^i(\mathcal{F} \otimes \mathcal{L}, \mathcal{G}) \cong \text{Ext}^i(\mathcal{F}, \mathcal{L}^\vee \otimes \mathcal{G}) \cong \text{Ext}^i(\mathcal{F}, \mathcal{G}) \otimes \mathcal{L}^\vee$$

and so

$$(\text{global}) \text{Ext}^i(\mathcal{F} \otimes \mathcal{L}, \mathcal{G}) \cong \text{Ext}^i(\mathcal{F}, \mathcal{L}^\vee \otimes \mathcal{G})$$

⁷This is because $\text{Ext}^i(\mathcal{O}_X, -)$ is the right derived functor of $\text{Hom}(\mathcal{O}_X, -)$, which is the identity functor. By contrast, $\text{Hom}(-, \mathcal{O}_X)$ is not the identity functor, and in general its right derived functor $\text{Ext}^i(-, \mathcal{O}_X)$ is nonzero for $i > 0$.

In [7] it was noted that particle-antiparticle duality was realized by Serre duality, a fact which we expect to continue to hold even for non-locally-free sheaves. For locally free sheaves \mathcal{E} , Serre duality on an n -dimensional projective Cohen-Macaulay⁸ variety is the statement

$$H^i(\mathcal{E}) \cong H^{n-i}(\mathcal{E}^\vee \otimes \omega)^\vee \quad (6)$$

where ω is the dualizing sheaf (equal to the canonical bundle when the variety is smooth). For more general coherent sheaves the correct statement⁹ of Serre duality on a projective Cohen-Macaulay variety is [22, section III.7]

$$\text{Ext}^i(\mathcal{O}_X, \mathcal{E}) \cong \text{Ext}^{n-i}(\mathcal{E}, \omega)^\vee$$

and in particular on a Calabi-Yau variety this is simply

$$\text{Ext}^i(\mathcal{O}_X, \mathcal{E}) \cong \text{Ext}^{n-i}(\mathcal{E}, \mathcal{O}_X)^\vee \quad (7)$$

We should point out that this correction solves a problem associated with any attempt to hypothesize (0,2) mirror symmetry for torsion-free sheaves. In principle, if X, Y are two Calabi-Yau's with sheaves \mathcal{E}, \mathcal{F} , respectively, such that each pair defines an anomaly-free heterotic conformal field theory, then if $(X, \mathcal{E}), (Y, \mathcal{F})$ are (0,2) mirror pairs then

$$\dim \text{Ext}_X^i(\mathcal{E}, \mathcal{O}_X) = \dim \text{Ext}_Y^i(\mathcal{O}_Y, \mathcal{F})$$

or, equivalently (by Serre duality),

$$\dim \text{Ext}_X^i(\mathcal{O}_X, \mathcal{E}) = \dim \text{Ext}_Y^i(\mathcal{F}, \mathcal{O}_Y)$$

Now, if we did not know that sheaf cohomology groups should be replaced with Ext groups, and incorrectly applied the duality in equation (6), then the analogous two statements

$$\begin{aligned} \dim H^i(X, \mathcal{E}^\vee) &= \dim H^i(Y, \mathcal{F}) \\ \dim H^i(X, \mathcal{E}) &= \dim H^i(Y, \mathcal{F}^\vee) \end{aligned}$$

would only be equivalent if \mathcal{E} and \mathcal{F} were both reflexive sheaves. In general, we would have two inequivalent possible formulations of (0,2) mirror symmetry. By recognizing that

⁸ Examples of Cohen-Macaulay varieties include toric varieties [18] and Calabi-Yau's in dimensions less than 4 [19].

⁹ The technically astute reader will recognize this as a special case of the Yoneda pairing

$$(\text{global}) \text{Ext}^p(\mathcal{F}, \mathcal{G}) \otimes \text{Ext}^q(\mathcal{G}, \mathcal{H}) \rightarrow \text{Ext}^{p+q}(\mathcal{F}, \mathcal{H})$$

valid for all coherent \mathcal{F}, \mathcal{G} , and \mathcal{H} . In fact, for the special case of coherent sheaves \mathcal{E}, \mathcal{F} on a smooth projective variety, Serre duality can be generalized [20, 21] to the statement

$$(\text{global}) \text{Ext}^i(\mathcal{E}, \mathcal{F}) \cong \text{Ext}^{n-i}(\mathcal{F}, \mathcal{E} \otimes \omega)^\vee$$

massless modes are counted with Ext groups rather than sheaf cohomology groups, this ambiguity is resolved.

In passing we should probably mention a subtlety associated with the Dirac index, which counts the generation number in heterotic compactifications. Strictly speaking, the Dirac operator is only defined when coupled to a locally free sheaf, because only then can we define a connection and thereby make sense out of $D = \partial + A$. As the Dirac operator is only defined when coupled to a locally-free sheaf, strictly speaking we can only define the Dirac index when coupled to a locally free sheaf. So, how can we compute the generation number when working with more general torsion-free sheaves? Instead of computing the Dirac index to find the generation number, one can also count particles directly. In general by counting chiral fermions one finds that the generation number is of the form

$$\sum_i (-)^i \dim \operatorname{Ext}_X^i(\mathcal{E}, \mathcal{F})$$

for torsion-free sheaves \mathcal{E} and \mathcal{F} . The Riemann-Roch theorem for general coherent sheaves [24] is

$$\sum_i (-)^i \dim \operatorname{Ext}_X^i(\mathcal{E}, \mathcal{F}) = \langle X | ch(\mathcal{E})^* \cup ch(\mathcal{F}) \cup td(TX) \rangle$$

where by $ch(\mathcal{E})^*$ we mean that all cohomology elements of degree $4n + 2$ (for some n) in $ch(\mathcal{E})$ should be multiplied by -1 . (For example, if \mathcal{E} were a bundle, then $ch(\mathcal{E})^* = ch(\mathcal{E}^\vee)$.) In particular, we recover the result for the generation number that one would have naively guessed. For example, for a compactification on a threefold X involving a torsion-free sheaf \mathcal{E} with $c_1(\mathcal{E}) = 0$, the generation number is proportional to $\langle X | c_3(\mathcal{E}) \rangle$, formally identical to the result for the special case \mathcal{E} is a bundle.

5 Conclusions

In this paper we have corrected two common misconceptions regarding perturbative compactifications of heterotic string theory. We have demonstrated that the usual statement of the anomaly freedom constraint is incorrect in general, and we have also corrected a common miscounting of massless particles in heterotic string theory.

Far more work remains to be done before our understanding of heterotic compactifications begins to approach our understanding of type II compactifications. At this point the physics community does not even have a good understanding of classical results, much less any understanding of quantum corrections. Significant progress in understanding the classical physics will be reported in [13].

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